Teaching materials

Guide notes 3. Acceleration analysis of a four-bar mechanism

MISCE project

Mechatronics for Improving and Standardizing Competences in Engineering

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1 Objective

The main objective of this lecture is to strengthen students' understanding of planar kinematics trough the acceleration analysis of a four-bar mechanism.

The session focuses on determining the angular accelerations of the moving links using known position and velocity data, assuming a constant input speed. Students will apply analytical methods to solve the acceleration problem and validate their results through experimental measurements collected from the four-bar mechanism platform.

2 Context

The mechanical system employed is a four-bar mechanism driven by a DC-motor. The input signal to this motor is a pulse width modulated (PWM) signal sent and controlled via a MATLAB-based control app (see Lesson 0. Introduction to the experimental platform).

This activity allows students to determine the angular accelerations of the mechanism's links based on known positional and velocity data. Building upon the outcomes of the position and velocity analyses, the lesson helps students understand how accelerations propagate through closed-loop linkages. Ultimately, it offers valuable hands-on experience in comparing theoretical acceleration models with real-world measurements.

3 Acceleration analysis of a four-bar mechanism

The goal of the acceleration analysis is to determine the angular acceleration α_4 of the rocker link, given the known positions and velocities of all the links and assuming a constant angular velocity for the crank ($\omega_2 = 1$ rad/s, thus $\alpha_2 = 0$). Although the input link has zero angular acceleration, the other links may still experience acceleration due to the geometry and motion of the mechanism.

The analysis begins by differentiating the position vector loop equation twice:

$$\ddot{\boldsymbol{r}}_2 + \ddot{\boldsymbol{r}}_3 = \ddot{\boldsymbol{r}}_1 + \ddot{\boldsymbol{r}}_4$$

Since the fixed link ($\boldsymbol{r}_1$) is constant, the equation simplifies to:

$$\ddot{\boldsymbol{r}}_2 + \ddot{\boldsymbol{r}}_3 = \ddot{\boldsymbol{r}}_4$$

Using the complex form of the acceleration vectors and assuming constant link lengths (so that $\dot{\rho}_i = 0$ and $\ddot{\rho}_i = 0$), the equation becomes:

$$(\cancel{\ddot{\rho}_2} - \omega_2^2\rho_2)e^{i\theta_2} + (\alpha_2\rho_2 + 2\omega_2\cancel{\dot{\rho}_2})ie^{i\theta_2} + (\cancel{\ddot{\rho}_3} - \omega_3^2\rho_3)e^{i\theta_3} + (\alpha_3\rho_3 + 2\omega_3\cancel{\dot{\rho}_3})ie^{i\theta_3} =$$
$$= (\cancel{\ddot{\rho}_4} - \omega_4^2\rho_4)e^{i\theta_4} + (\alpha_4\rho_4 + 2\omega_4\cancel{\dot{\rho}_4})ie^{i\theta_4}$$

Rewriting and grouping terms:

$$-\omega_2^2\rho_2 e^{i\theta_2} + \alpha_2\rho_2 ie^{i\theta_2} - \omega_3^2\rho_3 e^{i\theta_3} + \alpha_3\rho_3 ie^{i\theta_3} = -\omega_4^2\rho_4 e^{i\theta_4} + \alpha_4\rho_4 ie^{i\theta_4}$$

Splitting into real and imaginary parts yields a linear system in terms of the unknowns α_3 and α_4:

Real part:

$$-\omega_2^2\rho_2\cos(\theta_2) - \alpha_2\rho_2\sin(\theta_2) - \omega_3^2\rho_3\cos(\theta_3) - \alpha_3\rho_3\sin(\theta_3) = -\omega_4^2\rho_4\cos(\theta_4) - \alpha_4\rho_4\sin(\theta_4)$$

Imaginary part:

$$-\omega_2^2\rho_2\sin(\theta_2) + \alpha_2\rho_2\cos(\theta_2) - \omega_3^2\rho_3\sin(\theta_3) + \alpha_3\rho_3\cos(\theta_3) = -\omega_4^2\rho_4\sin(\theta_4) - \alpha_4\rho_4\cos(\theta_4)$$

These equations can be rearranged into matrix form and solved for α_3 and α_4 using Cramer's rule:

$$\begin{bmatrix} -\rho_3\sin(\theta_3) & \rho_4\sin(\theta_4) \\ \rho_3\cos(\theta_3) & -\rho_4\cos(\theta_4) \end{bmatrix} \begin{Bmatrix} \alpha_3 \\ \alpha_4 \end{Bmatrix} = \begin{Bmatrix} \rho_2\omega_2^2\cos(\theta_2) + \rho_2\alpha_2\sin(\theta_2) + \rho_3\omega_3^2\cos(\theta_3) - \rho_4\omega_4^2\cos(\theta_4) \\ \rho_2\omega_2^2\sin(\theta_2) - \rho_2\alpha_2\cos(\theta_2) + \rho_3\omega_3^2\sin(\theta_3) - \rho_4\omega_4^2\sin(\theta_4) \end{Bmatrix}$$

Additionally, the angles either come from known values (like θ_2) or from the position analysis, and the velocities have been previously calculated during the velocity analysis.

4 Methodology

4.1 Example: Solving the acceleration problem

For the example in this document, the following geometrical values for the four-bar mechanism's links are given:

- $\rho_1 = 0.14$ m
- $\rho_2 = 0.05$ m
- $\rho_3 = 0.16$ m
- $\rho_4 = 0.1$ m

It is also assumed that $\omega_2 = 1$ rad/s, $\alpha_2 = 0$ rad/s^2, the angular positions θ_2, θ_3 and θ_4 are known from the position analysis, and the angular velocities ω_3 and ω_4 are known from the velocity analysis.

Solving the linear system for each value of θ_2 gives:

Table 1. Angular acceleration α_4 for $\omega_2 = 1$ rad/s and $\alpha_2 = 0$ rad/s^2

θ_2 (°)	α_4 (rad/s^2)
45	0.5989
135	-0.1805
225	-0.3459
315	-0.5384

When using a constant velocity input, the angular acceleration α_4 follows a typical cyclic pattern, just like in the velocity analysis. On the other hand, if the velocity input is uniformly accelerated, this periodicity disappears: the magnitudes of acceleration increase, and the motion cycles shorten progressively.

4.2 Student task

In the experiment, students will rotate the motor driving the crank (link 2) and record the angular acceleration of the rocker (link 4) using its encoder. Note that the link lengths should be predefined by the student. Then, they will analytically solve the problem to obtain the output angular velocity α_4, as explained in the theoretical lessons.

The student will repeat the experiment for a uniformly accelerated input velocity.

By comparing analytical results with experimental data, they can verify the accuracy of their acceleration analysis.